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Dark energy in perturbative string cosmology

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Abstract: The apparent observation of dark energy poses problems for string theory. In de Sitter space, or in quintessence models, one cannot define a gauge-invariant S-matrix. We argue that eternal quintessence does not arise in weakly coupled string theory, but point out that it is difficult to define an S-matrix even in the presence of perturbative potentials for the moduli. The solutions of the Fischler-Susskind equations all have Big Bang or Big Crunch Singularities. We believe that an S-matrix (or S-vector) exists in this context but cannot be calculated by purely perturbative methods. We study the possibility of metastable de Sitter vacua in such weakly coupled scenarios, and conclude that the S-matrix of the extreme weak coupling region cannot probe de Sitter physics. We also consider proposed explanations of the dark energy from the perspective of string theory, and find that most are implausible. We note that it is possible that the axion constitutes both the dark matter and the dark energy.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

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Contents

1. Introduction

Accelerating universes are not compatible with the conventional setup of string theory. The appearance of a cosmological horizon in many models incorporating rolling scalar fields signals the absence of a completely gauge invariant S-matrix [1, 2, 3]. This poses a phenomenological challenge for string theory, since our universe appears to be accelerating [4]. One may ask however if it can lead us to an inconsistency of the string theoretic formalism itself. That is, supersymmetric vacua of string theory have exactly massless moduli fields at tree level. There are a variety of situations in which we believe that we can break SUSY, either at tree level, or through low energy non-perturbative effects, in a controllable manner. That is, SUSY breaking generates a potential for the moduli, which attracts the system to the weak coupling regime, where the cosmological constant vanishes. One can then imagine setting up a scattering theory in which we choose a solution of the effective equations of motion in which the dilaton is in the extreme weak coupling regime both in the infinite past and the infinite future. The asymptotic states are then those of freely moving string excitations, and we can imagine computing the S-matrix for scattering from some free particle state in the past to a different one in the future. Fischler and Susskind [5] have provided a prescription that purports to obtain well defined vertex operator correlation functions in the case that the potential is generated in perturbation theory. These are presumably the S-matrix elements in question. We will use the phrases Fischler-Susskind Cosmology and Cosmological S-matrix (FSC and FSCS-matrix) to describe both perturbative and non-perturbative scenarios of this kind.

The papers of [2, 3] pose a potential problem for this program. They show that in many models with rolling scalar fields there is a cosmological horizon and no sensible S-matrix exists. In this paper we will show, following old work of Brustein and

Steinhardt [6] that this problem does not occur in FSC. No plausible FSC solution has a horizon. However, we will point out that all such solutions have singularities of the Big Bang or Big Crunch type. Thus, it is *not* true that we have a controllable weak coupling string calculation in this scenario.

Our analysis is done at the level of low energy effective field theory and it is barely possible that α' corrections eliminate the singularity. We argue that this is unlikely to be true, by using the holographic principle.

Nonetheless, we believe that in all of these situations, an S-matrix or S-vector [1, 7] exists, though we may need the full non-perturbative definition of the theory to describe the initial state (in the case of a Big Bang).

This leads us to another question. Suppose that, by utilizing a variety of fluxes [8], one succeeds in calculating a potential for moduli that stabilizes them in a regime where string perturbation theory is apparently accurate. Suppose further that the value of the potential at the non-trivial minimum is non negative. How does one describe the properties of the stable or metastable state at this minimum in terms of the original weakly coupled string theory from which the potential was derived. That is, ignoring the problems with the Big Bang singularity, can one set up an initial state that will probe the properties of the non-trivial minimum?

We argue, using the results of [9, 10] that this is unlikely to be the case. These papers argued that in an asymptotically Minkowski spacetime, generic attempts to access a DeSitter or isolated Minkowski minimum, lead instead to the formation of a black hole. The same black hole can be created in ways that have nothing to do with the non-trivial minima (the No Hair theorem). According to Black Hole Complementarity [11], physics as seen by observers inside the black hole is described by operators (including the time evolution operator) that do not commute with any of the observables at infinity. Thus there is no way for the scattering matrix to encode information about physics at the non-trivial minimum, as it would be experienced by an observer who thought that he/she was in DeSitter space.

Although FSC spacetimes are not asymptotically flat, they are, in Einstein frame, Friedmann-Robertson-Walker (FRW). Thus, the same conclusions can be drawn in this case (modulo lingering uncertainties about the meaning of initial conditions at the Big Bang). The transformation between Einstein and string frames is asymptotically singular in the weak coupling regime (and both metrics are singular at the Big Bang). The proper framework for any attempt to construct a perturbative S-matrix is the string frame, because the string frame metric is what appears in the vertex operator construction of scattering amplitudes. Nonetheless, we argue that the conclusions about the FSCS-matrix that we draw from the Einstein frame analysis are completely valid.

Stepping away from the larger conceptual issues that are the main subject of this paper, we examine a number of proposals for the dark energy. We conclude that existing proposals require phenomena far different than any known in string theory, or remarkable coincidences beyond the cancellation of the cosmological constant and the near equality of the dark matter and dark energy densities. We discuss the proposal of [12], arguing that it is consistent with the observed facts. We also note that the QCD axion could plausibly constitute both the dark matter and the dark energy.

2. No cosmological horizons in perturbative string theory

Some years ago, Brustein and Steinhardt [6] argued that the potentials expected from SUSY breaking scenarios in weakly coupled string theory could not give rise to inflation, nor indeed to any asymptotically accelerating expansion of the universe. The argument was given in the context of a single scalar field, and is easy to recapitulate. We will work in Einstein frame because that is the frame in which horizon areas are a measure of entropy.

The equations of motion of a minimally coupled (canonically normalized) scalar field and gravity, assuming a $d+1$-dimensional, flat FRW universe, are:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{2\kappa^2}{d(1-d)} E \equiv \frac{\dot{\phi}^2}{2} + V(\phi)\,, \qquad \ddot{\phi} + dH\dot{\phi} + V'(\phi) = 0\,, \tag{2.1}$$

where $\kappa^2 = 8\pi G_N$ is the gravitational coupling. Assuming an expanding universe, these can be written as a single equation for E as a function of ϕ:

$$E_\phi = -2\sqrt{\frac{d}{d-1}}\sqrt{2E(E-V)}\,. \tag{2.2}$$

We are in a situation where the potential is positive, and asymptotes to zero at $\phi = \infty$. The question of whether there is a cosmological horizon depends only on how fast V asymptotes to zero. To see this, note that the coordinate of the horizon is given by

$$R_H = \int \frac{dt}{a} = \int d\phi \left[\frac{1}{a\dot{\phi}}\right] = \int d\phi \left[a\sqrt{2(E-V)}\right]^{-1}\,. \tag{2.3}$$

The scale factor, a, is given by:

$$a = \exp\left[\int d\phi \sqrt{\frac{E}{d(d-1)(E-V)}}\right]\,. \tag{2.4}$$

If the energy is asymptotically dominated by the potential, then a will diverge at large ϕ, much more rapidly than $E \to V$. Then the integral defining R_H will be finite for all times, and there is a cosmological horizon. Nothing beyond the coordinate distance R_H will be visible to a local observer.

Now suppose V vanishes more rapidly than an exponential. We claim that $E \gg V$ asymptotically. Indeed, neglecting V in (2.2) we see that E vanishes exponentially and the approximation is self-consistent. Conversely, if V vanishes less rapidly than an exponential then so must E. Thus E must asymptote to V, $E = V + \Delta$, with

$$\Delta_\phi \simeq -2\sqrt{\frac{2d}{d-1}}\sqrt{V\Delta}\,. \tag{2.5}$$

Consider finally the case where $V = e^{-\alpha\phi}$ and define $E \equiv rV$. Then:

$$r_\phi - \alpha r = -2\sqrt{\frac{d}{d-1}}r\sqrt{(1-r^{-1})} \tag{2.6}$$

with the condition $r \geq 1$. This equation has a fixed point, which automatically obeys the bound on r. Otherwise, its asymptotic behavior is dominated by large r. In either case, $E = Ce^{-\beta\phi}$ with

$$\beta = \pm 2d\sqrt{\frac{d}{d-1}} \tag{2.7}$$

when $r \to \infty$ and

$$\beta = \alpha\,, \tag{2.8}$$

for the fixed point solution. It is easy to see that there will be a horizon only for the fixed point case, and only if $2/\alpha(d-1) > 1$.

The result of Brustein and Steinhardt is now easy to understand. In string theory, the coupling is an exponential of a canonically normalized dilaton field. Thus, any non-perturbative potential will be an exponential of an exponential and there will be no horizon. For a potential generated at one loop, we have to do a bit more work, to figure out the exponent in terms of the canonical dilaton in Einstein frame. However, this is easy and even the one-loop potential violates the bound on α implied by the existence of a horizon. In ten dimensions, for example, if the potential is generated at one loop, $\alpha = 5/4\sqrt{2}$. In four dimensions, the situation is more complicated, since there are typically several moduli. In an asymmetric orbifold compactification with no geometric moduli, or in other compactifications with all of the moduli but S frozen, $g^2 = e^{\phi/\sqrt{2}}$, and a one-loop potential would have $V = cg^2$. This is an exponential of the dilaton vanishing more rapidly than the bound.

One may question whether this result is truly general. The tachyon free SUSY violating heterotic string in ten dimensions provides an example with only a dilatonic modulus. More general FSC models will have many moduli. There is a metric on moduli space and the free motion of homogeneous modes on the moduli space is chaotic [13]. Furthermore, the coupling of the homogeneous and inhomogeneous modes produces an instability for production of the latter [14].

Fortunately, none of these questions affect the existence of a horizon, since it is a phenomenon which occurs only asymptotically, and only if the motion of the moduli is, in the end, dominated by the potential energy. Because of the potential, even the chaotic system will eventually reach the asymptotic region (remember, according to the ground rules of FSC models, the potential is everywhere positive and has no minima except at infinity). Furthermore, the instability of [14] shuts off for small velocity and will not affect the late stages of motion if there is a horizon.

Now note that the Friedmann equation and the equation for E as a function of time, $\dot{E} = -dH\sqrt{2(E-V)}$, are still valid, with E given by $E = \frac{1}{2}\dot{M}^i G_{ij}(M)\dot{M}^j + V(M)$. Any solution of the equations of motion for the moduli, will follow some trajectory through moduli space. Define the path length variable $Z = \int dt\sqrt{\dot{M}^i G_{ij}\dot{M}^j}$. Then $E = \frac{1}{2}\dot{Z}^2 + V(M(Z))$. We now have a problem of a single variable again, with Z playing the role of ϕ. So we need only know the behavior of V along the asymptotic trajectory in moduli space, as a function of the path length.

Asymptotic directions in moduli space correspond to $d+1$-dimensional coupling going to zero (because we are talking about FSC models), perhaps combined with a blowup of some internal dimensions. Models with blowing up internal dimensions are harder to analyze, because the asymptotic effective field theory has a larger number of dimensions than the initial theory. For trajectories where blowup does not occur, our previous analysis holds. We have only cursorily examined other models and found no examples with horizons, but have stopped short of an exhaustive study because we are not completely sure how to interpret the S-matrix when some dimensions become large asymptotically.

3. Bangs and crunches

Saving the universe from quintessence has turned out to be a simple task in FSC models. Saving it from the singularity theorems of Penrose and Hawking [15] is quite another matter. Indeed, asymptotically, all FSC models approach $p = \rho$ FRW universes, since the kinetic energy dominates the potential. Thus, they contain a singularity either in their past or their future (note that the d spatial dimensions are not compactified and we cannot use dualities to remove singularities). This might not be so bad if the singularity of the asymptotic solution in the future, was in the past, while that of the asymptotic past solution was in the future. Then we could hope that the complicated intermediate dynamics somehow removed the singularity. This cannot happen.

The desired state of affairs requires expansion in the remote future and contraction in the remote past. But the Friedmann equation states that H can change sign only at zeroes of the energy, and the energy can only vanish if $\phi = \infty$ and $\dot{\phi}$ approaches zero there (remember again that the philosophy of FSC models requires us to remain in regions where the asymptotic form of the potential is valid).

We can try to fix this up with a positive spatial curvature. Now H can vanish for positive energy. It is easy to see that if the curvature term is large enough when H vanishes then contraction truly turns into expansion. Unfortunately, there is a fly in the ointment. Asymptotically, the field energy vanishes like a^{-2d} while the curvature falls only like a^{-2}. Thus, we can have neither contraction from infinite size in the past nor expansion to infinite size in the future. The universe undergoes a Big Crunch, and began with a Big Bang.

After contemplating these disasters for a few minutes, one realizes that one was "doomed from the start". Our models all satisfy the conditions of the Hawking-Penrose singularity theorems. The generality of those results leads us to conclude that there is no escape within the realm of low energy effective field theory. Are there stringy loopholes?

3.1 Can we blame the frame?

Since our solutions evolve to regions of weak coupling, we can phrase the problem of solving the string equations in terms of solving the β-function equations for a two-dimensional conformal field theory. One potentially mitigating feature of these equations is that they are most naturally written, not in the Einstein frame, but in the string frame. In the string frame, the curvatures are smaller than in the Einstein frame by a power of the coupling constant. In this section, we will see, however, that even the string frame curvature is too singular to permit a perturbative solution of the beta function equations. We will see that this feature holds independent of the detailed form of the potential (e.g. whether it arises at one or two loops, or non-perturbatively). Thus if the problem has a solution, it cannot be found perturbatively in the α' expansion. Conceivably, one can find an exact conformal field theory which is non singular. It is interesting that, because the asymptotic behavior of the curvature (and the dilaton) in the singular region is independent of the potential, this problem is likely to either have no solution at all, or a solution which is universal.

To determine the behavior of the potential, it is helpful to look at the solutions of the (Einstein frame) equations in more detail. Consider first the case of ten dimensions, and suppose we consider a theory such as one of the ten-dimensional, non-supersymmetric, non-tachyonic string theories, which develops a potential at one loop.

In terms of the canonically normalized dilaton field, D, the potential has the form:

$$V = e^{-\alpha D} \tag{3.1}$$

while the coupling is given by

$$g^{-2} = e^{-\gamma D}\,. \tag{3.2}$$

One can determine the asymptotic behavior of the fields by means of the procedure outlined above. For the case of an expanding universe, one finds:

$$t \longrightarrow -\infty :\ a \longrightarrow e^{\sqrt{1/d(d-1)}\phi} \tag{3.3}$$

so the curvature, in the Einstein frame, behaves as

$$\mathcal{R} = -d(1-d)\left(\frac{\dot{a}}{a}\right)^2 \left(\frac{2}{d(d-1)} e^{\sqrt{d/d-1}\phi}\right). \tag{3.4}$$

On the other hand, the coupling behaves, in ten dimensions, as

$$g^2 = e^{-\phi/\sqrt{2}}. \tag{3.5}$$

So even in the string frame, the curvature blows up. Again, note that the problem is universal; it does not depend on the details of the potential. In other dimensions, the results are similar. For example, in four dimensions, if we freeze the volume modulus, g behaves identically, whereas $d = 3$ in the formula for the behavior of a.

We conclude that the Big Bang singularities of FSC solutions cannot be removed by the conformal transformation to string frame. It is conceivable that exact conformal field theories could be found, which removed the singularity found at lowest order in the α' expansion. However, the results of [16] suggest that this is not the case. These authors studied Kasner solutions of M-theory on the moduli spaces with 16 or more supercharges. It was found that every solution contained a singular region which could not be dual transformed into a weakly coupled string theory, smooth eleven-dimensional SUGRA, or even an F-theory type regime. In the singular region, the FSB [17] bounds suggest that the Hilbert space describing physics inside a particle horizon has a small finite dimension, which shrinks as one approaches the singularity. It seems unlikely that any weakly coupled string theory could describe such a situation.

Although these results suggest that singularities cannot be removed at string tree level, the question remains an interesting one and deserves further study.

Finally, we would like to suggest an hypothesis about the relevance of perturbative string calculations to physics in the FSC background. String theory is an S-matrix theory, and as such appears to describe infinite numbers of incoming and outgoing initial states. The Fischler-Susskind prescription formally preserves this property. On the other hand, we have suggested that all solutions of the FS equations have at least a Big Bang singularity. It has been suggested that there may be a unique initial state at such a singularity. One way in which such a conclusion might arise self consistently within the stringy formalism is that most scattering amplitudes calculated by the FS prescription would simply diverge. One would then discover that the divergences vanished only for a particular choice of initial state (and presumably only after a resummation of the perturbation expansion).

3.2 Metastable DS and isolated M vacua

There has been much recent interest in string models which stabilize all moduli at values where string perturbation theory might be valid. A basic idea is that non-zero Ramond-Ramond fluxes on cycles of the compactification manifold, and Dbranes wrapped on such cycles, give contributions to the energy that scale as different powers of the string coupling. By contemplating large fluxes, one can stabilize the dilaton at weak coupling. It is harder to stabilize the volume of the compactification manifold , and in fact the best that has been achieved so far is to generate a no scale model in the SUGRA approximation [8].[1] Higher order corrections will give a potential for the volume modulus, which vanishes at infinite volume. Perhaps the large fluxes will appear in this potential in a way that gives it a minimum at a value where systematic calculations are possible.

Similarly, there are perturbative string compactifications on asymmetric orbifolds, which freeze all the geometrical moduli, leaving only the dilaton. One can imagine e.g. racetrack scenarios in which a calculable minimum is found at weak coupling. Again, the potential will vanish at asymptotically weak coupling.

In all such models we have two candidate background geometries for string theory. The first is the FSC solution we have been discussing in this paper, in which the modulus starts infinitely far away, rolls up the hill of the potential and rolls back again. Although this solution contains a Big Bang singularity, we have argued that it should be described by a well defined S-vector. Although this does not allow us to freely specify initial conditions, one can certainly imagine that, because the barriers between the non-trivial minimum and the state at infinite modulus are parametrically smaller than the Planck scale, there is a finite probability to push the system into the non-trivial minimum in some local region in space. The question now is how such an event manifests itself in the scattering amplitudes.

The value of the potential at its minimum is clearly an important determinant of what happens. If it is negative, there is an instanton [18] that describes decay of the FSC into a stable AdS minimum. Since we are assuming both the vacuum energy and the barrier between the AdS minimum and infinity are parametrically smaller than the Planck scale (defined in terms of the minimum value the Planck scale of the non-compact dimensions attains in the FSC solution), this instanton is below the Coleman-DeLuccia bound and the decay actually occurs.[2] Furthermore, the expansion of the universe in the FSC solution is subluminal, so vacuum bubbles collide and percolate. There is, strictly speaking, no FSC state of the system, which is rather described by an AdS vacuum of string theory. By the AdS/CFT correspondence, this suggests the existence of a non-supersymmetric conformal field theory with the rather peculiar pattern of operator dimensions that are necessary to describe a large

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[1] This statement does not take into account as yet unpublished material. J. Polchinski and E. Silverstein have independently informed us of forthcoming work in which stable, non-supersymmetric AdS vacua of weakly coupled string theory are found. E. Silverstein has also proposed a more speculative scenario for describing a metastable DS minimum. The remarks which follow were written before we learned of these new results.

[2] The natural scale of variation of the potential for the moduli is the string scale, parametrically smaller than the Planck scale. This is important to the conclusion about the CDL bound.

radius AdS space. If the string coupling at the AdS minimum is small, one imagines this CFT to be a gauge theory with relatively large-N and large 't Hooft coupling. An interesting "inverse question" arises: is it possible to see evidence for a metastable FSC state in the large-N gauge theory?

If the value of the potential at the minimum is positive, we are close to the situation investigated by Guth and Farhi [9]: we are attempting to create a bubble of DeSitter universe in our FSC background. If we work in Einstein frame this situation resembles that of Guth and Farhi so much that their conclusion follows. The stress tensor satisfies the dominant energy condition. The analysis of black hole formation is essentially local, and at least in the late stages of the FSC cosmology, when the universe is expanding slowly, it is unaffected by the general cosmological expansion. Indeed, Guth and Farhi intended their analysis to apply to the real world, which is a Robertson-Walker cosmology and not an asymptotically flat universe. The FSC cosmology differs from that of the real world only by the equation of state of the dominant matter at late times.

The conformal factor relating the string frame to the Einstein frame is singular only asymptotically, when the string coupling goes to zero in the infinite past and future. The Guth-Farhi black hole is formed locally, at a time when the string coupling is finite. Thus, again, we cannot invoke the transformation to string frame to attempt to avoid the conclusions of their analysis.

We now turn to the case of an hypothetical, isolated asymptotically flat vacuum which is calculable. Namely, we assume that by inserting a number of large fluxes, one stabilizes all moduli including the dilaton, at a value where string perturbation theory is applicable and the vacuum energy is exactly zero. Needless to say, there are no known examples of vacua of this type. One may be interested in them in two different contexts: the first is conventional string phenomenology, where such a vacuum is presumed to be non supersymmetric, and corresponds to the real world. The conjectures of [12] deny this possibility but postulate the existence of a supersymmetric isolated vacuum state toward which the theory of the real world would asymptote if the cosmological constant were taken to zero. In either case one must address the interesting practical question of whether perturbative calculations in string theory can have any relevance for the real world. This could be the case if the isolated vacuum occurred at weak string coupling because of the existence of large topological invariants like fluxes.

As above, these scenarios exhibit two classical background spacetimes. The first is the FS cosmology at asymptotically weak coupling, while the second is the isolated asymptotically flat vacuum. We have to ask which of these is stable, and whether one can detect the unstable one inside the stable one. The stability of the isolated vacuum toward decay into the FSC solution seems clear. If we use the symmetries of the FSC solution it would seem that the only possible way to compare the two backgrounds is to match the cosmological time of the FSC solution to the Minkowski time in some Lorentz frame. At any finite cosmological time, the FSC solution has positive energy density, and there is no instanton that allows the asymptotically flat spacetime to decay into it. In the case of an exactly SUSY vacuum, stability follows from SUSY. SUSY violating, asymptotically Minkowski vacua can sometimes exhibit semiclassical instability [19], but they decay into "nothing", rather than into a positive energy cosmology.

The question of stability of the FSC solution is more subtle. For simplicity of exposition we will restrict attention to four dimensions, though the generalization to arbitrary dimensions is easy. If we neglect the cosmological expansion, it is clear that, given the presumed parametrically small potential for the dilaton, there are bubbles of isolated vacuum whose growth is energetically favored. Let ϵ be the instantaneous energy density difference between the Minkowski and FSC solutions and σ the instantaneous surface tension of a bubble separating them. Then, neglecting numbers of order one, the critical bubble size is σ/ϵ. Assuming both σ and ϵ are much smaller than the Planck scale, we would normally presume such bubbles to expand with the speed of light. Since the cosmological expansion of the FSC background is subluminal we might then expect percolation of the bubbles and complete decay of the FSC cosmology into flat spacetime.

This analysis neglects the time dependence of the parameters of the bubble. The energy density difference ϵ is constantly decreasing with time. Further, if we consider the late time evolution, when the dilaton is moving toward the weak coupling regime and away from the isolated minimum, then σ is increasing with time. The critical bubble size is thus increasing rapidly with time at late times. It makes no sense to discuss a critical bubble of size larger than the cosmological horizon. At any time the universe can be viewed as made up of decoupled quantum systems, which describe physics inside disjoint backward lightcones whose tips lie on that time slice. Thus, we must have $\sigma/\epsilon < M_P/\sqrt{\epsilon}$. As above, M_P is defined in terms of the value of the string coupling at the turnaround point, and does not vary with time. Bubble nucleation must surely stop once this inequality fails, and this is inevitable as the universe expands.

Indeed, it is very likely that the bubble nucleation process is dominated by events which occur near the point of turnaround, when the dilaton reaches its maximal height on the potential. Earlier on, the expansion rate of the universe is much larger than the bubble nucleation rate and the Coleman-DeLucia analysis [18] that we have been using is inapplicable. The horizon size is very small. It is reasonable to presume that few if any bubbles are nucleated during this period. Near the turnaround point the instanton action is at its minimum: the energy density is small compared to the Planck scale and is of the same order of magnitude as the surface tension , while the classical FSC configuration is as close as it gets to the isolated minimum. Within the realm of validity of the semiclassical analysis, the probability per unit time per unit volume for bubble nucleation will be very small.

Now consider the expansion of these bubbles. The cosmological increase in the tension of their walls and simultaneous decrease of ϵ will act to slow the expansion. It seems likely to us, though we have as yet no proof, that the asymptotic expansion rate is likely to be slower than the speed of light. The bubble thus becomes visible to a distant observer, and will have (if it continues to expand) a radius of order $R > \sigma/\epsilon$ and mass of order $\sigma_{\mathrm{eff}} R^2$. Here σ_{eff} is an effective surface tension. One might guess a formula $\sigma_{\mathrm{eff}} \sim \sigma\sqrt{1-v^2}$ where v is the asymptotic speed of the bubble expansion. It is clear that if the radius gets too large, the bubble will be inside its own Schwarzchild radius and will collapse into a black hole. Thus, there are bounds on the bubble size of order $\sigma/\epsilon < R < M_P^2/\sigma_{\mathrm{eff}}$. It is clear that σ_{eff} must increase as the universe expands. It is proportional to σ, which grows, and the velocity certainly should not increase as the universe expands. Thus, eventually the bubble must recollapse, and the FSC solution is stable. We are aware that this argument is far from rigorous and that we have not completely ruled out the possibility of decay of the FSC solution into isolated vacuum, but we believe it is implausible.

We will now briefly discuss the question of whether finite energy processes in either the isolated Minkowski or FSC backgrounds can create large metastable bubbles of the other solution, which could be explored by experimentalists living in one of these alternative universes. We begin with the asymptotically Minkowski background, which has a well defined S-matrix. A bubble of FSC solution will have finite energy, parametrically smaller than the Planck scale. In the setup we are imagining, the barriers between the FSC regime and the Minkowski vacuum are also parametrically small. Thus, there will be a range of bubble sizes for which the bubble is larger than its Schwarzchild radius. As time goes on inside the bubble, the dilaton decreases. This would tend to increase the barrier between the FSC and Minkowski solutions, and therefore must contribute to accelerated collapse of the bubble. However, given our assumptions, we can tune the rate of these processes to be slow by tuning large fluxes. Finally note that since the expansion inside the bubble is subliminal, there is no paradox in assuming that an observer dropped into the bubble can report back to his colleagues outside about the processes going on there.

Similar remarks apply to creation of a small bubble of Minkowski vacuum in the FSC cosmology. There is a small philosophical difference. If we truly believe in the notion of an S-vector rather than an S-matrix, we have to accept the (surely approximate) notion of the free will of local observers in order to claim with 100% probability that such experiments can actually be done. If the time evolution of the universe unfolded uniquely from a unique initial state then one would only have to hope that there was a sufficiently large probability that a bubble creation event occurred. This is to be contrasted to the Minkowski situation, where an infinite set of initial conditions is part of the mathematical setup. Again, with the parameters as we are assuming them, the bubble creation events are sufficiently localized and occur at sufficiently low energies that these unpalatable philosophical questions are probably not important.

At any event, we are not really interested in the bubble creation experiments, which are totally impractical even if not ruled out in principle. The key issue is whether, in the situation we are hypothesizing, the mathematical apparatus of perturbative string theory, which (apart from times near the Big Bang singularity) describes the physics of the FSC solution, can be used to calculate properties of the isolated Minkowski vacuum. The above considerations suggest that this is indeed the case, at least with some limited accuracy.

4. What the dark energy isn't, and what it might be

Much of this paper has been devoted to a demonstration that accelerated expansion does not occur in controllable situations in string perturbation theory, and to an exploration of the perturbative physics that does occur. In this section we briefly discuss stringy perspectives on the problem of Dark Energy.

A number of proposals have been made for the dark energy. While we can hardly claim to understand what the dark energy might be in string theory, many of these proposals seems implausible. There are a variety of difficulties. Some have to do with the required scales; some are related to the problem of horizons. Still others have to do with issues peculiar to possible anthropic explanations.

Among the proposals are a variety having to do with brane pictures. In scenarios with large but finite extra dimensions, standard effective field theory arguments indicate that one will inevitably obtain too large a cosmological constant. In theories with infinite extra dimensions, the effective theory arguments do not immediately apply, but the various proposals lead to singularities, whose interpretation is at best unclear. We have nothing further to add on this question here.

We have already noted the arguments of [2, 3] that quintessence, if it is eternal, leads to horizons which are problematic in string theory. We will note further difficulties with quintessence below, associated with the required scales.

One of the most puzzling aspects of the dark energy problem is the question of coincidence: why is the scale of the dark energy today so close to that of the dark matter? At the moment, the most plausible explanations of this fact are anthropic. Inevitably, any successful anthropic explanation of the cosmological constant problem will predict a dark energy density within an order of magnitude or so of the dark matter density [20, 21]. At least two classes of anthropic explanations have been widely discussed recently. The first requires the presence in the theory of a large number of possible four form fluxes. The different discrete values of these fluxes then lead to a large number of metastable states with a "discretum" of energies. There are a number of difficulties with this proposal, which are discussed in [22]. In particular, in these schemes, one needs to suppose that there are a vast number of non-supersymmetric (metastable) states. In string theory we have yet to reliably exhibit one. Perhaps more fundamental is the problem that in these proposals not only is the cosmological constant determined anthropically, but all of the other parameters of the standard model are either anthropic or random variables. But this seems unlikely. While we might imagine that anthropic considerations would determine the masses of the light quarks and leptons, for example, it is less plausible that such considerations determine the heavy quark masses and mixings. These parameters hardly appear random.

The second class of anthropic proposals requires the presence of an extremely light scalar, with Compton wavelength of order the size of the present horizon or larger. The idea is that the value of this field is essentially a random variable during inflation. Different parts of the universe will have different values of the cosmological constant depending on the value of the field in that region. If, for example, the potential is $m^2\phi^2$, then this can cancel a negative cosmological constant, say of order $10^8\,\mathrm{GeV}^4$, provided that ϕ is large enough.

In string theory, however, it seems implausible that a field so light can carry so much energy. How, first, might we imagine getting such a light field? There is no evidence that in string theory, scalar fields are appreciably lighter than the scale of supersymmetry breaking, in the absence of a symmetry. More precisely, there are many situations where we can study supersymmetry breaking in a controlled approximation. In these cases, all fields gain mass of order the supersymmetry breaking scale, $M_{\rm susy}$, or perhaps $M_{\rm susy}^2/M_p$. This is much the same as occurs in supergravity theories.[3]

The only examples of symmetries which might yield such light scalars are axions. Now the mass of an axion might plausibly be of order

$$m_a^2 = e^{-8\pi^2/g^2}\frac{M_{\rm susy}^4}{M_p^2}\,, \tag{4.1}$$

where we might imagine that g is of order some typical unified coupling, and supposed that the axion decay constant is within a few orders of magnitude of the Planck mass. We might also imagine that $M_{\rm susy} \sim 10^{10}\,\mathrm{GeV}$. This would give

$$m_a \sim 10^{-33}\,\mathrm{GeV} \tag{4.2}$$

which gives a Compton wavelength not wildly different than the size of the universe (it is about three orders of magnitude smaller). Given the huge uncertainties in this estimate, this is an interesting result.

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[3] Note that in the case of moduli, in known, controlled examples, the moduli have non-trivial potentials; by mass we mean the second derivatives of the potential, where appropriate.

In any case, the potential for such an axion is periodic, with period approximately f_a, and one does not expect that it will be larger than

$$V_o = e^{-8\pi^2/g^2} M_{\rm susy}^4 \qquad (4.3)$$

i.e. it will be of order 70 orders of magnitude smaller than the expected contributions to the vacuum energy from supersymmetry violation.

It is interesting, on the other hand, that this crude estimate is in the right ballpark for the axion itself to provide the dark energy. There are two ways this might happen. First, we might postulate that, in addition to the axion which explains the smallness of the QCD theta parameter, there is another axion, with mass of order the mass given by this estimate. Then this axion might still be frozen at a point away from its minimum, and the observed dark energy could just be this stored energy [23, 27]. For this to be the case, however, it is important that the axion energy density should be of order the observed energy density, while the mass is small. This is problematic. It requires an additional coincidence: in order that the axion not be rolling now (so that it's equation of state will resemble that of a cosmological constant) it is necessary that its mass be smaller than the present Hubble constant. But this mass is related to the energy density by (assuming that the axion energy is a simple cosine, i.e. $V = C\cos(af_a)$, or similar periodic function)

$$m_a^2 f_a^2 = \left(\frac{3}{8\pi}\right) H^2 M_p^2 \,, \qquad (4.4)$$

where a_o is the present value of the axion field. So even if the axion decay constant is as large as the (reduced) Planck mass, the axion compton wavelength will not be larger than the present horizon. So in effect, we now have two coincidences: the potential is just such that it dominates the energy density during the current epoch, and the axion decay constant is just such that the axion is about to roll, but hasn't quite begun yet.[4]

So it seems unlikely that the explanation of the dark energy is that there is an axion sitting near the top of a hill. This requires a particle present solely for this purpose, with both energy density and decay constant (mass) tuned just so.

Similar remarks apply to quintessence, which also requires a field with a Compton wavelength comparable to the present horizon, but whose energy density must be comparable to the present energy density. Once more, in the absence of a symmetry, we know of no example in string theory where the scale of the potential of a particle is not related to the scale of supersymmetry breaking, without some additional, Peccei-Quinn like symmetry. Difficulties with axion-like particles as quintessence, beyond

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[4] Note, for example, that in the Horava-Witten picture, the axion decay constant is several orders of magnitude below the Planck mass; in weakly coupled string theory, it is of order the string scale, which is suppressed by a factor of coupling relative to the Planck scale. It seems unlikely that one can obtain a decay constant much larger than the Planck scale.

those described above, have been discussed in [26, 27]. The former reference outlines in some detail the special circumstances required to obtain axion domination now. It points out difficulties with fields other than the axion as quintessence. The latter discusses observational difficulties associated with a quintessence axion.

There is a possible alternative role for axions, which doesn't require the addition of a particle solely for the purpose of explaining the dark energy, and which requires only one, not totally implausible, coincidence. Consider the ordinary QCD axion. As has frequently been discussed in the axion literature, if the coefficient of the QCD anomaly is not $1/16\pi^2$ but $N/16\pi^2$, then the QCD contribution to the axion potential takes the form

$$V = m_\pi^2 f_\pi^2 \cos(aNf_a)\,. \tag{4.5}$$

In this case, there are N degenerate ground states. This degeneracy holds exactly in QCD, and reflects the fact that QCD breaks the original PQ symmetry down to a Z_N. More precisely, it holds in the limit that

- Only effects connected with the anomaly break the Peccei-Quinn symmetry.
- The SU(2) gauge coupling is set to zero.

Once we turn on the SU(2) gauge coupling, the Z_N symmetry may be broken by SU(2) instantons. This will be the case if the anomalous coupling of the axion to SU(2) is different than to SU(3) (say 1 instead of N). Considerations of the low energy, renormalizable theory might suggest that these contributions will receive additional suppression, involving many Yukawa couplings and loop factors. These are necessary to tie together the many fermion zero modes. But in general it should be possible to tie up these zero modes with high dimension operators. Indeed, no symmetry (except, possibly, anomalous discrete symmetries) can forbid the appearance of high dimension operators with the quantum numbers of the 't Hooft operator itself under any approximate low energy symmetries. These operators will be suppressed by powers of the large scale (e.g. the unification scale). This just means that the principle contribution to the amplitude will come from very small instantons. The real suppression lies in the exponential of the gauge coupling, but by using the unified coupling in our estimates, we have taken this into account. Additional suppression factors, such as powers of π etc. depend on the details of the theory.

More generally, then, we might expect, due to instantons at the unification scale, M, symmetry breaking effects of order

$$e^{-8\pi^2/g^2(M)} M_{\rm susy}^4 \tag{4.6}$$

and, as we have seen, this is a number easily within a few orders of magnitude of the observed dark energy density! In other words, we might imagine that in the lowest energy state, for (mysterious) reasons, the cosmological constant vanishes; then there are a set of nearly degenerate states, with an energy density of order that which is observed![5]

Of course, in this view, the cosmic coincidence seems to be an accident, with a chance of order one part in a thousand, or perhaps smaller. We would note, however, that this is no worse than another, somewhat more vague suggestion for understanding the coincidence [25]. Some authors have noted that that the observed dark energy density is very nearly the fourth power of $\mathrm{TeV}^2/\tilde{M}_p$, where $\tilde{M}_p$ is the reduced Planck mass. They have argued that this is a plausible form for a microscopic expression for the energy density, given that TeV is of order the weak scale. Indeed, this gives a result within a factor of 10 of the observed density. On the other hand, writing the formula as

$$\Lambda = \frac{(c\ \mathrm{TeV})^8}{M_p^4} \tag{4.7}$$

makes clear that there are many orders of magnitude uncertainty even in this crude estimate. For example, if c is 3; this would increase the answer by a factor of almost 10^4! So we would claim our proposal is as good (or bad) an explanation of the cosmic coincidence as any other non-anthropic proposal.

Finally we mention the proposal of [12], to which one of us must confess a certain attachment. In this proposal, there is a true cosmological constant. Furthermore, it is assumed to be a fundamental input parameter, rather than a calculable quantity in the low energy effective action. The reasoning is that the cosmological constant, according to the holographic principle, measures the total number of states in the Hilbert space describing the universe. In quantum mechanics, the total number of states is always a fixed boundary condition, rather than a dynamical quantity.

From this point of view, the puzzle of the actual value of Λ would be resolved only by anthropic reasoning. There could be a Meta-theory that produces some probability distribution for the size of the Hilbert space describing a particular universe[6]

The other possibility is that the number of states N has to satisfy some number theoretic identity whose solutions are very sparse. At first sight it would appear that the number $e^{10^{123}}$ is so huge that it is hard to believe this possibility. On the other hand, there are problems in number theory which have no or only a few

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[5] As this paper was being completed, we received the paper [24] which also argues that the lifting of the N-fold axion degeneracy might account for the dark energy. In this proposal, it is argued that operators of very high dimension might break the PQ symmetry by a tiny amount, small enough not to spoil the solution of the strong CP problem, but large enough to account for the vacuum energy.

[6] We are quite uneasy about the prospect of such a theory. It describes the probabilities for alternative universes and in principle can never be tested in our own. Furthermore, as long as the probability distribution it produces has non-zero support in the anthropic region and is not peaked at one of the extremes of that region it will be compatible with observations. Thus, few if any of the details of this theory have any effect on any observable quantity.

known solutions, but no proof to date that there is no other.[7] Perhaps, for some peculiar reason, the number of states has to be an odd perfect number. Another possibility, which does not rely on an unproven mathematical conjecture is that the number of states has to be of the form 2^p where p is a Mersenne prime (a prime of the form $2^k - 1$). There are only two values of k, 521 and 607, in the table of Mersenne primes which would give a cosmological constant within shouting distance of the true value (and both are off by many orders of magnitude) according to this formula. Thus, although the particular example of Mersenne primes does not work, it is easy to imagine number theoretic criteria that would allow only one value of the number of states which was in any way realistic. Systems with vastly smaller numbers of states could not exhibit any sort of interesting physics, while those with vastly larger numbers of states are likely to be described by low energy physics that is superconformally invariant down to extremely low energy scales [12].

In the end, the number theoretic option still has to resort to the anthropic principle. However, (assuming that it is easy to rule out life in a superconformal world) the anthropic arguments are much simpler. For all solutions except one the world would either be described by a system with a number of states too small to support complex systems, or a system that was superconformal down to incredibly low energy scales.

Of course the real challenge for this set of ideas is verification of the claim that SUSY breaking scales with an unconventional power of Λ as $\Lambda \to 0$.

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